M2 Jan 2009 Written Solutions

① $\int v \, dt = \int 4t^3 - 8\sin 2t + 5 \, dt$

$$s = t^4 + \frac{8}{2}\cos 2t + 5t + c = t^4 + 4\cos 2t + 5t + c$$

$s = 0$ when $t = 0$ $\Rightarrow$ $0 = 0 + 4\cos(0) + 0 + c$

$0 = 4 + c \Rightarrow \underline{\underline{c = -4}}$

$s = t^4 + 4\cos 2t + 5t - 4$

② (a) $E_k = \frac{1}{2}mu^2 = \frac{1}{2} \times 6 \times 12^2 = 432\text{ J}$

(b)(i) As it rises $E_k \to GPE$. As it then falls $GPE \to E_k$ so since we ignore work done against air resistance, when it reaches its <u>initial position</u> it will have 432 J of $E_k$ again

Final $E_k$ = $\underbrace{\text{initial } E_k}_{432}$ + $\underbrace{\text{loss in GPE}}_{mg \times 4}$

$$\frac{1}{2}mv^2 = 432 + 6 \times 9.8 \times 4$$
$$= 667.2$$

$$\Rightarrow v = \sqrt{\frac{2 \times 667.2}{6}} = 14.913$$

$$\Rightarrow \underline{\underline{v = 14.9\text{ ms}^{-1}}}$$

3)(a) $\underline{v} = \frac{d\underline{r}}{dt} = \left(\frac{1}{2} \times 2e^{\frac{1}{2}t} - 8\right)\underline{i} + (2t - 6)\underline{j}$

$= \left(e^{\frac{1}{2}t} - 8\right)\underline{i} + (2t - 6)\underline{j}$

)(i) $\underline{v} = \left(e^{\frac{1}{2} \times 3} - 8\right)\underline{i} + (2 \times 3 - 6)\underline{j}$

$= \left(e^{\frac{3}{2}} - 8\right)\underline{i} + 0\underline{j}$

$\Rightarrow$ velocity $= e^{\frac{3}{2}} - 8 = -3.518$ $\Rightarrow$ speed $= 3.52\text{ ms}^{-1}$

(ii) remember → $\underline{j}$ component was 0.

it is in the negative $\underline{i}$ direction ⇒ **West**.

(c) $t = 3$ $\quad a = \frac{d\underline{v}}{dt} = \left(\frac{1}{2}e^{\frac{1}{2}t}\right)\underline{i} + 2\underline{j}$

$$\Rightarrow a = \left(\frac{1}{2}e^{\frac{3}{2}}\right)\underline{i} + 2\underline{j}$$

(d) $m = 7\text{kg}$ $\quad F = ma$

$$= 7\left(\frac{1}{2}e^{\frac{3}{2}}\underline{i} + 2\underline{j}\right)$$

$$= \frac{7}{2}e^{\frac{3}{2}}\underline{i} + 14\underline{j}$$

Magnitude $= \sqrt{\left(\frac{7}{2}e^{\frac{3}{2}}\right)^2 + 14^2} = 21.0\text{N}$

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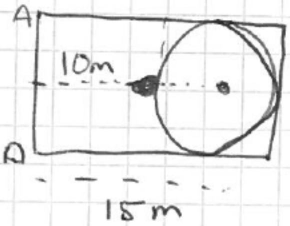
(a)



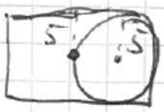

x-coordinate only

Total mass $= 8 + 2 = 10\text{kg}$

$$10\bar{x} = \underbrace{15 \times 2}_{\text{from circle com}} + \underbrace{10 \times 8}_{\text{from rectangle com}}$$

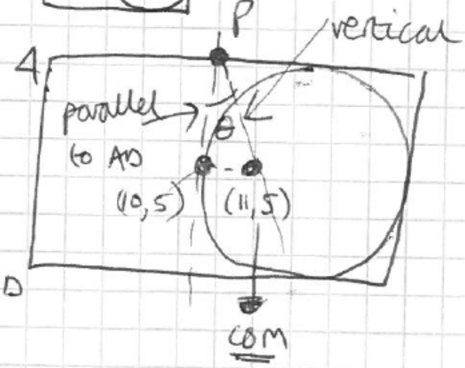
$\Rightarrow 10\bar{x} = 110 \quad \Rightarrow \underline{\bar{x} = 11\text{cm}}$

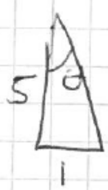
b)




Must be $\underline{5\text{cm}}$

c)




$\Rightarrow \tan\theta = \frac{1}{5}$ and $\theta = \tan^{-1}\left(\frac{1}{5}\right)$

$$= 11.3^\circ$$

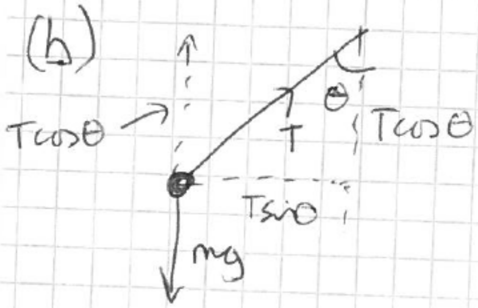
(d) COM is in the centre of the individual shapes (circle & rectangle).

⑤ (a) 40 rev min$^{-1}$

$40 \times 2\pi = 80\pi \Rightarrow w = 80\pi$ rad min$^{-1}$

so $\div 60$ $\quad w = \frac{4}{3}\pi$ rads$^{-1}$

(b)




resolve vertically

$T\cos 30 = mg$

$T = \frac{6g}{\cos 30} = 67.896$

$\Rightarrow T = 67.9$ N

(3SF)

(c) Net force $= T\sin\theta$

Use $F = ma \Rightarrow T\sin\theta = mrw^2$

$67.896 \times \sin 30 = 6\left(\frac{4\pi}{3}\right)^2 r$

$r = \frac{67.896 \times \frac{1}{2}}{6 \times \frac{16\pi^2}{9}} = 0.322$ m

⑥ $m = 60$ tonnes $= 60000$ kg $\quad P = 800000$ W

$v = 40$ ms$^{-1}$

(a) $r \leftarrow \bullet \rightarrow$ driving force $F_D$ $\quad F_D = \frac{P}{v} = \frac{800000}{40}$

$= 20000$ N

max speed $\Rightarrow a = 0$

$\Rightarrow$ Net force $= 0$ and so $r = F_D$

resistive force $= 20000$ N

(b) $40$ ms$^{-1}$ $\rightarrow 36$ ms$^{-1}$

$\Delta E_k = \frac{1}{2}m(40^2 - 36^2) = 30000 \times 304 = 9120000$ J

$\Rightarrow$ This must be "lost" as work done against resistive force

$wd = F \times s \quad \Rightarrow s = \frac{wd}{F} = \frac{9120000}{20000} = 456$ m

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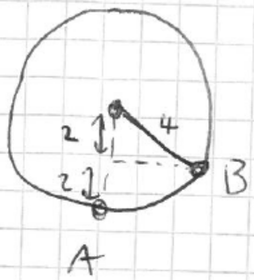



(a) $E_k$ at $A = \frac{1}{2} \times 6 \times 8^2 = 1$

Gain in GPE from A to B $= mg\Delta h$

$= 6g \times 2$

$= 117.6 \text{ J}$

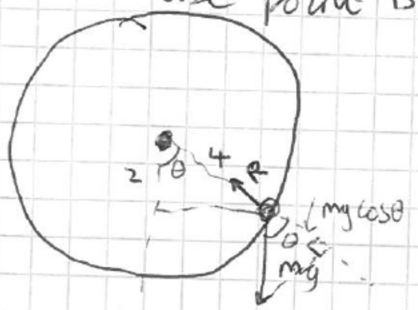
So $E_k$ at $B = 192 - 117.6 = \underline{\underline{74.4 \text{ J}}}$

$\frac{1}{2}mv^2 = 74.4 \implies v = \sqrt{\frac{74.4 \times 2}{6}} = \sqrt{24.8}$

$= 4.97996$

$\underline{v = 4.98 \text{ ms}^{-1}}$

(b) Use point B




$\cos\theta = \frac{1}{2}$

$\implies \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60°$

Consider component of mg acting radially $= mg\cos\theta$

Net force (radially) $= R - mg\cos\theta$

$= R - 6g\cos 60$

$= R - 3g$

Use $F = ma = \frac{mv^2}{r}$

$R - 3g = \frac{6 \times 24.8}{4}$ ← $v^2$ at $B = 24.8$

$\implies R - 3g = 37.2$

So $\underline{\underline{R = 66.6 \text{ N}}}$

(8) $m = 0.05\,kg \qquad F = -0.08v^2$

(a) $a = \frac{F}{m} = \frac{-0.08v^2}{0.05} = -1.6v^2$

so $a = \frac{dv}{dt} = -1.6v^2$

(b) $\frac{dv}{dt} = -1.6v^2$

Separate variables $\Rightarrow \int \frac{1}{v^2}\,dv = -\int 1.6\,dt$

$\int v^{-2}\,dv = -\int 1.6\,dt$

$\Rightarrow \quad -v^{-1} = -1.6t + c$

$\frac{-1}{v} = -1.6t + c$

$(\times -1)$

$\frac{1}{v} = 1.6t + d$

when $t = 0 \quad v = 3$

$\Rightarrow \quad \frac{1}{3} = 0 + d \quad \Rightarrow \quad d = \frac{1}{3}$

so $\frac{1}{v} = 1.6t + \frac{1}{3}$

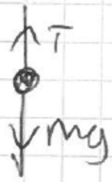
$v = \frac{1}{1.6t + \frac{1}{3}} \times \frac{15}{15}$

$= \frac{15}{24t + 5}$

So $v = \frac{15}{5 + 24t}$ as required.

(9) $l = 16 \qquad \lambda = 784N$

(a)




$\underline{\underline{a=0}} \Rightarrow F = 0$

so $T = mg$

$$T = \frac{\lambda x}{l} = \frac{784x}{16}$$

$T = mg$ so $\frac{784x}{16} = 80 \times 9.8$

$$x = \frac{80 \times 9.8 \times 16}{784} = 16m$$

Extension $= 16$ m  so length $= l + e$

$= 16 + 16 = \underline{\underline{32m}}$

(b) It means that it has extended $x$ when it comes to rest.

$EPE = \frac{\lambda x^2}{2l}$  and  $\Delta GPE = -mg(16+x)$

$= -80g(16+x)$

(loss in GPE)

GPE lost is converted into EPE.

so $80g(16+x) = \frac{\lambda x^2}{2l}$

$$12544 + 784x = \frac{784x^2}{2 \times 16}$$

$$12544 + 784x = \frac{49}{2}x^2$$

$\Rightarrow \quad 0 = \frac{49}{2}x^2 - 784x - 12544 \quad \div \frac{49}{2}$

$0 = x^2 - 32x - 512$  as required

(ii) $x^2 - 32x - 512 = 0$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{32 \pm \sqrt{32^2-4(1)(-512)}}{2 \times 1}$$

$$= 16 \pm \frac{\sqrt{3072}}{2} = 16 \pm 16\sqrt{3}$$

$16 + 16\sqrt{3}$ is when it first comes to rest

$(= 43 \cdot 71)$

So distance below start $= (16 + 16\sqrt{3}) + l$

$= 16 + 16\sqrt{3} + 16$

$= 32 + 16\sqrt{3}$

$= 59 \cdot 7128$

$65 - 59.7128 = 5 \cdot 287$

$= 5 \cdot 29\text{m}$

we need distance above ground